An-Najah National University
Faculty of Engineering

Mechanical Engineering Department
Control Systems I (67471)
Second Exam

Instructor: Dr. Nidal Farhat	**Student Name: .... Solution**
Credit Hours: 3	**Registration Number: ....**
Date: Thursday, April 09, 2015	**Total Exam Mark: 100**
Exam Duration: 50 min	**Exam Weight: 20**

Question	Points	ILO's	ILO's %	Question Grade
Q1	55	3	100	
Q2	45	3	100	
			Student Grade	

Exam Notes:

1. Solve all the problems.
2. Closed books and notes.
3. Read each problem carefully before attempting to solve it.
4. Write all work on this exam paper.

Instructors:

Dr. Bashir Nouri	Dr. Nidal Farhat
10:00 – 11:00	01:00 – 02:00

Question 1: (55 points)

For the following transfer function, and using **diagonal canonical form**, determine:

1) **The signal flow graph model**

$$T(s) = \frac{Y(s)}{R(s)} = \frac{4s + 12}{s^2 + 8s + 12}$$

$$T(s) = \frac{4s+12}{s^2+8s+12} = \frac{K_1}{(s+2)} + \frac{K_2}{(s+6)}$$

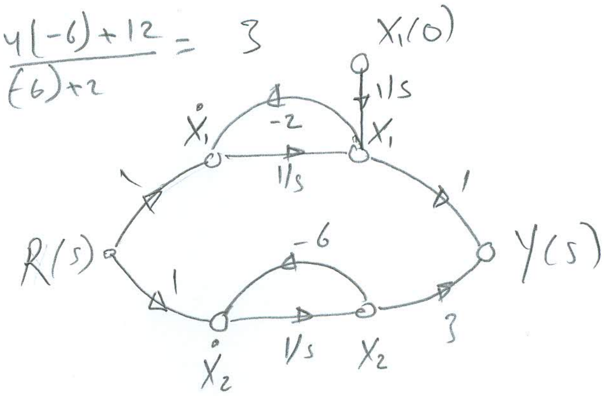
$$s_{1,2} = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times 12}}{2} = -4 \pm 2$$

(10)

$$K_1 = \frac{4s+12}{s+6}\Big|_{s=-2} = \frac{4(-2)+12}{(-2)+6} = 1$$

$$K_2 = \frac{4s+12}{s+2}\Big|_{s=-6} = \frac{4(-6)+12}{(-6)+2} = 3$$

(25)



(15)

2) **The state differential equation in matrix form.**

$$\dot{X}_1 = -2X_1 + r(t)$$

$$\dot{X}_2 = -6X_2 + r(t)$$

(10)

(5) marks on $\dot{X}_1$ & $X(s)$ on the S.F.G.

(20)

$$\dot{\vec{X}} = \begin{bmatrix} -2 & 0 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r(t)$$

(5)

3) $\Phi_{2,1}(t)$ from the signal flow graph

from $X_1(0)$ to $X_2(s)$

$$\phi_{2,1}(s) = \frac{X_2(s)}{X_1(0)} = \frac{\Sigma P_k \Delta_k}{\Delta}$$

(10)

from the S.F.G.M. $\Rightarrow$ No path from $X_1(0)$ to $X_2(s)$

$\Rightarrow P = 0$

$\Rightarrow \phi_{21}(s) = 0$ (Diagonal Canonical form)

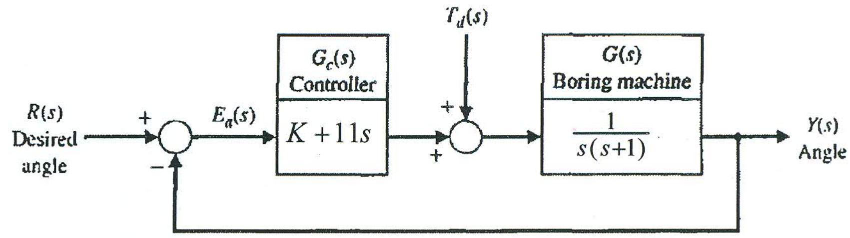
$\Rightarrow \underline{\phi_{21}(t) = 0}$

Question 2: (45 points)

For a system represented by the following block diagram model, determine:

(13)

1) Determine the sensitivity of the system to the change of the parameter K



$$S_K^T = \frac{\partial T}{\partial K}\frac{K}{T} \qquad T = \frac{\frac{K+11s}{s(s+1)}}{1+\frac{K+11s}{s(s+1)}} = \frac{K+11s}{s(s+1)+K+11s} = \frac{K+11s}{s^2+12s+K} \quad (5)$$

$$\frac{\partial T}{\partial K} = \frac{(1)(s^2+12s+K)-(K+11s)(1)}{(s^2+12s+K)^2} = \frac{s^2+s}{(s^2+12s+K)^2} \quad (5)$$

$$S_K^T \; (5) = \frac{s^2+s}{(s^2+12s+K)^2}\cdot\frac{K}{(K+11s)}\,\cancel{(s^2+12s+K)} = \frac{(s^2+s)K}{(s^2+12s+K)(K+11s)} = \frac{Ks^2+Ks}{11s^3+(132+K)s^2+23Ks+K^2}$$

(15)

2) The steady state error of the system to a unit step input.

[Unit feedback system]

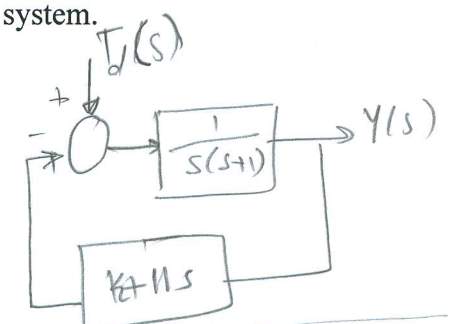
$$(5)\quad E_a(s) = R(s)-Y(s) = \frac{1}{1+GH(s)}R(s)$$

$$(5)\quad E_a(s) = \frac{1}{1+\frac{K+11s}{s(s+1)}}R(s) = \frac{\cancel{s}(s+1)}{s(s+1)+K+11s}\cdot\frac{1}{\cancel{s}} = \frac{s+1}{s^2+12s+K}$$

$$(5)\quad e_a(\infty) = \lim_{s\to 0} sE_a(s) = \lim_{s\to 0} s\left(\frac{s+1}{s^2+12s+K}\right) = \boxed{0}$$

(15)

3) The effect of unit step disturbance on the output response of the system.



(10)

$$Y(s) = \frac{\frac{1}{s(s+1)}}{1+\frac{1}{s(s+1)}\cdot(K+11s)}T_d(s)$$

$$= \frac{1}{s(s+1)+K+11s}T_d(s) = \frac{1}{s^2+12s+K}T_d(s)$$

$$(5)\quad y(\infty) = \lim_{s\to 0} s\left(\frac{1}{s^2+12s+K}\cdot\frac{1}{s}\right) = \frac{1}{K}$$

OR. $s_{1,2} = \dfrac{-12\pm\sqrt{(12)^2-4(1)(K)}}{2\times 1}$

(5) ⇒ Depending on the value of

$0 \le K < 36 \Rightarrow$ overdamped $\Rightarrow y(t)$

$36 < K \Rightarrow$ underdamped